Applications and Development status of HIMAG as a tool in liquid metal MHD problems in nuclear fusion

HyPerComp **I**ncompressible **M**HD solver for **A**rbitrary **G**eometry

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Funded by DOE SBIR Contracts monitored by Sam Berk and Gene Nardella

PFC Meeting, PPPL, May 9-11, 2005

A report of progress under the following SBIR contracts:

Phase-II: Practical simulations of two-phase MHD flows with wall effects

Contract # DE-FG02-04ER83676, August 2004 to July 2006

Phase-I: Development of a canonical approach to liquid metal MHD computations and experiments

Contract # DE-FG02-04ER83977, August 2004 to April 2005

Highlights of progress in the past 6 months

- B-field formulation and generic transport routines added to HIMAG
- Boiler-plate grid generation and problem setup for typical blanket geometries (channels, bends, manifolds)
- Canonical problem studies, numerical characterization and validation of the code
- Non-orthogonal corrections, div and curl cleanup for B-fields completed
- Turbulence (two equation, zero equation) models have been added – validation and code cleanup are ongoing
- Gradual additions are being made to HIMAG’s fusion portfolio

HIMAG

OVERVIEW
OF
CURRENT CAPABILITIES

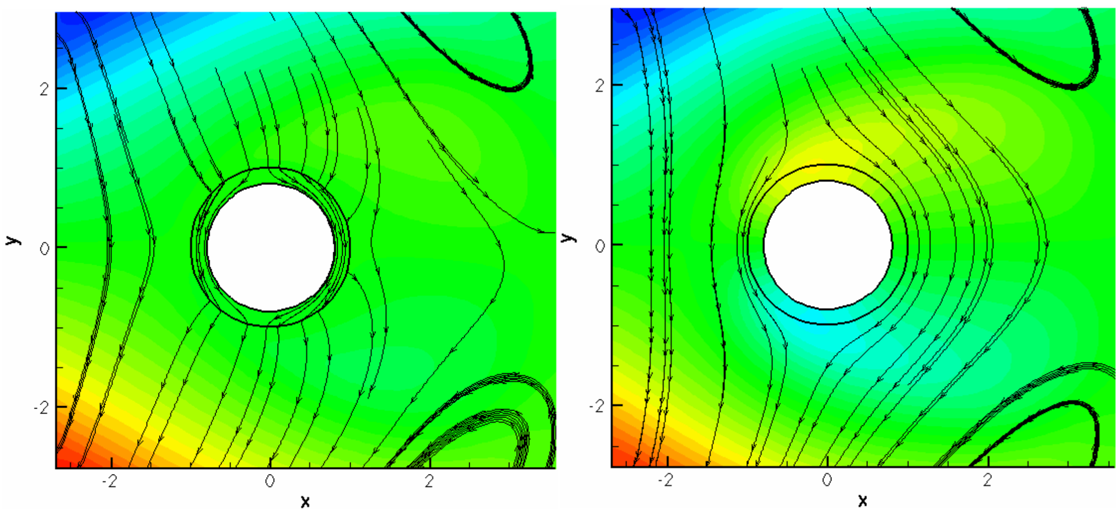
HIMAG attributes in a nut-shell

- HIMAG is based on a second order accurate algorithm for both time and space
- The code has been written for complex geometries and there is much flexibility in choosing a mesh: Hexahedral, Tetrahedral, Prismatic cells can be used.
- 2-D as well as 3-D flows can be simulated. A fully developed flow option has recently been created.
- The code is parallel. All problems can be run across parallel computers.
- Electromagnetics and flow physics are solved in a coupled manner. Two different formulations for electromagnetics are now available: phi and B.
- An arbitrary set of conducting walls maybe specified. Free surface flows are modeled using the Level Set method.
- Graphical interfaces are available to assist users from problem setup to post-processing.
- A preliminary turbulence and heat transfer modeling capability now exists.

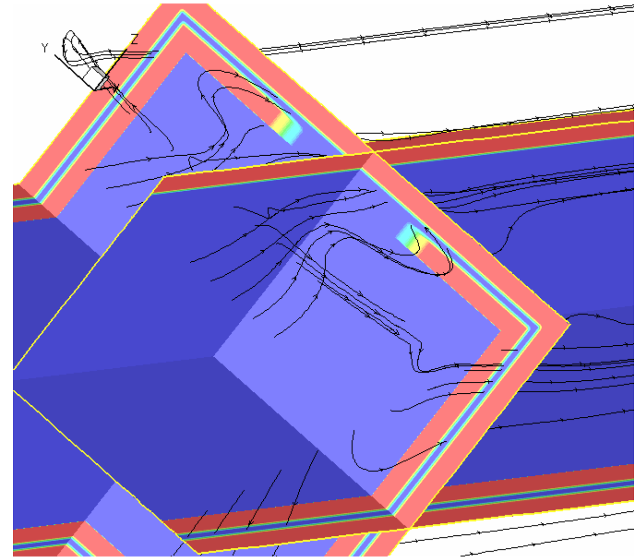
Closed channel flows

HIMAG is well suited to studying closed channel flows encountered in blanket conditions.

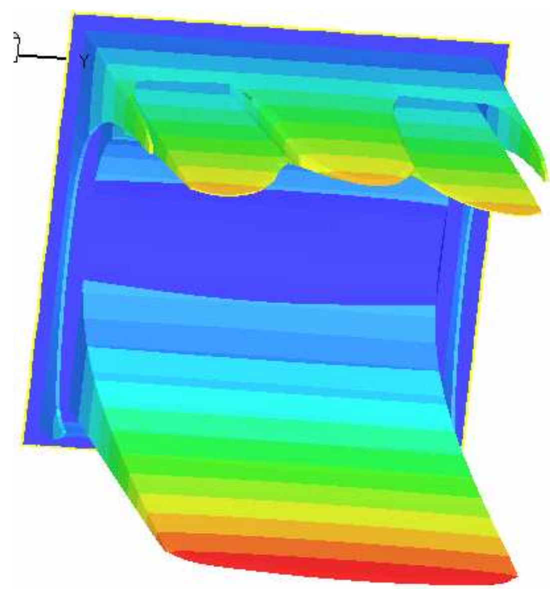
Complex geometric features can be resolved by using the flexible mesh structure, while high Hartmann numbers require the parallelism for quick turn around time.



Current in flow past a conducting and insulating cylindrical shell



Sample results from a 3-D FCI Pb-Li flow study. Streamlines (above), and velocity contours at a cross section (below)

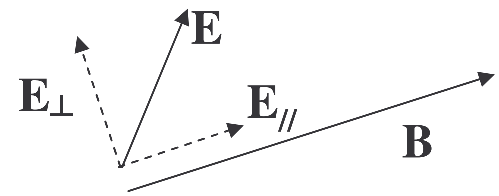


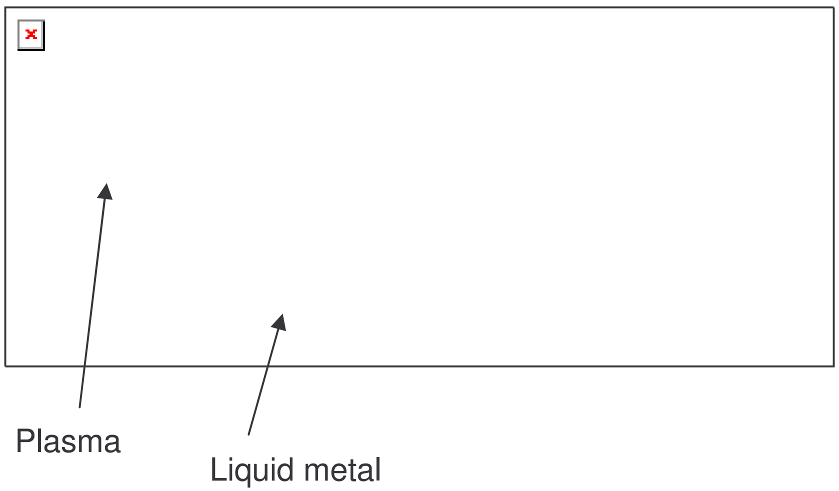
MHD Model in HIMAG for “plasma” current

Simplified Ohm’s law for anisotropic MHD

$$\vec{E}_{//} = \left(\vec{E} \cdot \frac{\vec{B}}{B} \right) \frac{\vec{B}}{B} \qquad \vec{E}_{\perp} = \vec{E} - \vec{E}_{//}$$

$$\begin{aligned}\vec{J} &= \sigma_1 \vec{E}_{\perp} + \sigma_2 \vec{E}_{//} \\ &= \sigma_1 \vec{E} + (\sigma_2 - \sigma_1) \vec{E}_{//} \\ &= \sigma_1 \left(-\vec{\nabla}\varphi + \vec{V} \times \vec{B} \right) + (\sigma_2 - \sigma_1) \left[\left(-\vec{\nabla}\varphi + \vec{V} \times \vec{B} \right) \cdot \frac{\vec{B}}{B} \right] \frac{\vec{B}}{B}\end{aligned}$$





Conservation form of div(J) = 0

$$\nabla \cdot \left(\sigma_{\perp} \nabla \varphi + (\sigma_{\parallel} - \sigma_{\perp})(\nabla \varphi \cdot \boldsymbol{b})\boldsymbol{b} \right) = \nabla \cdot \left(\sigma_{\perp} \mathrm{v} \times \mathrm{B} + (\sigma_{\parallel} - \sigma_{\perp})\left((\mathrm{v} \times \mathrm{B}^0) \cdot \boldsymbol{b} \right)\boldsymbol{b} \right)$$

Finite volume Poisson solver

$$\phi_P = \frac{\sum_{faces} \left(\frac{\sigma_{\perp}}{d} + \frac{(\sigma_{\perp} - \sigma_{//})(\boldsymbol{b} \cdot \boldsymbol{d})(\boldsymbol{b} \cdot \boldsymbol{n})}{d^2} \right) \phi_N \Delta s + \sum_{faces} \left(\sigma_{\perp} (\mathrm{v} \times \mathrm{B}^0) + \left((\sigma_{\perp} - \sigma_{//})(\mathrm{v} \times \mathrm{B}^0) \cdot \boldsymbol{b} \right) \boldsymbol{b} \right) \cdot \Delta \vec{s}}{\sum_{faces} \left(\frac{\sigma_{\perp}}{d} + \frac{(\sigma_{\perp} - \sigma_{//})(\boldsymbol{b} \cdot \boldsymbol{d})(\boldsymbol{b} \cdot \boldsymbol{n})}{d^2} \right) \Delta s}$$

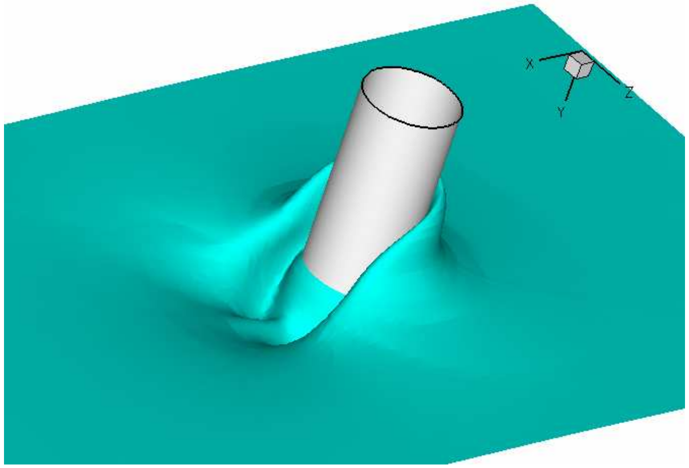
Plasma Momentum model could be instituted in tandem

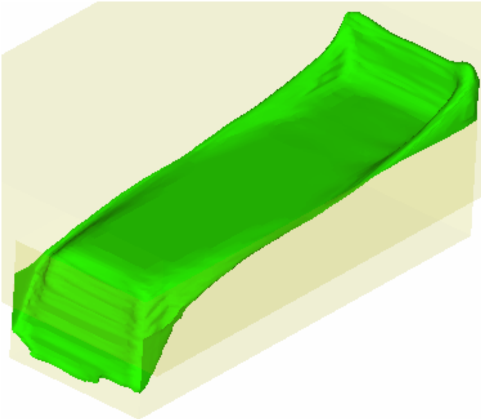
Free surface capture

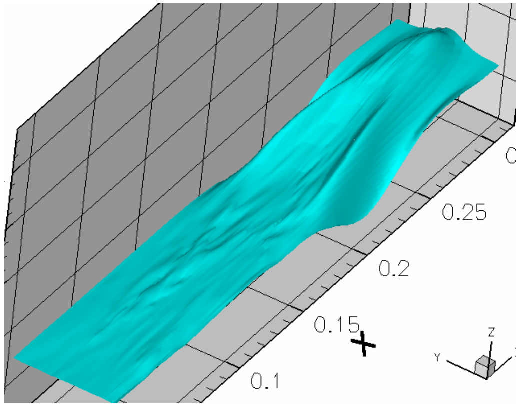
HIMAG uses the Level Set method to capture free surfaces

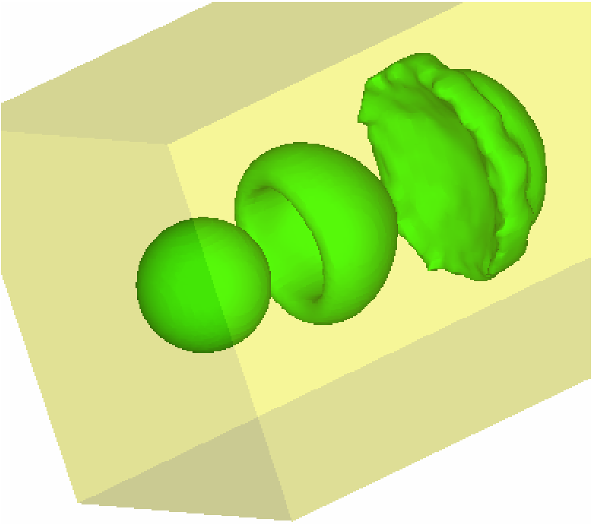
A higher order TVD procedure is used to enhance mass conservation and numerical stability

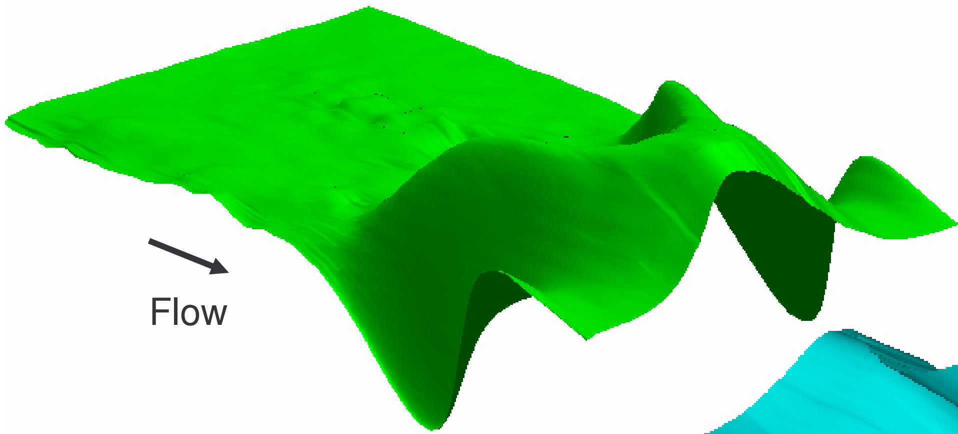
There is a constantly increasing portfolio of applications to problems relevant to fusion, as in the following charts





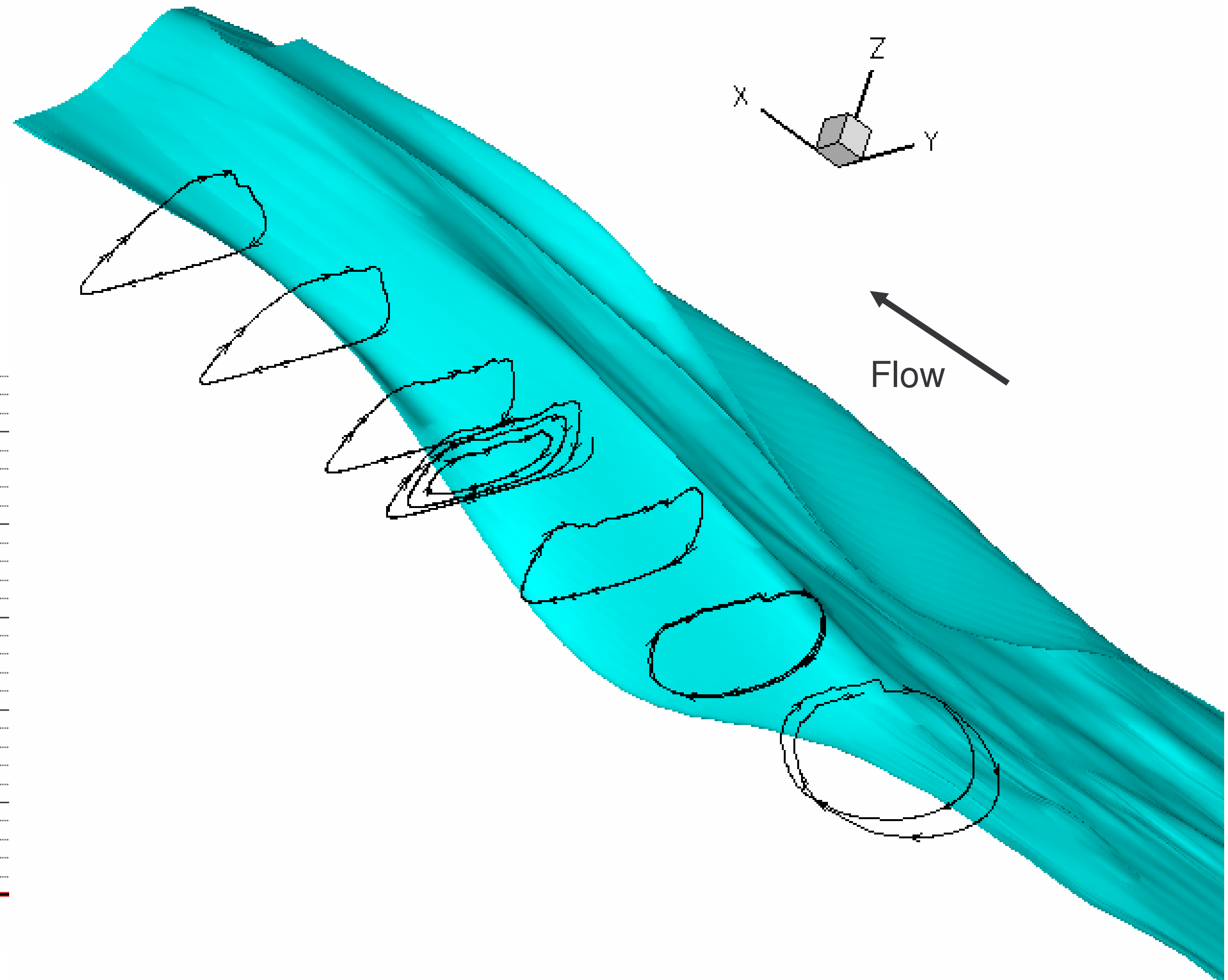


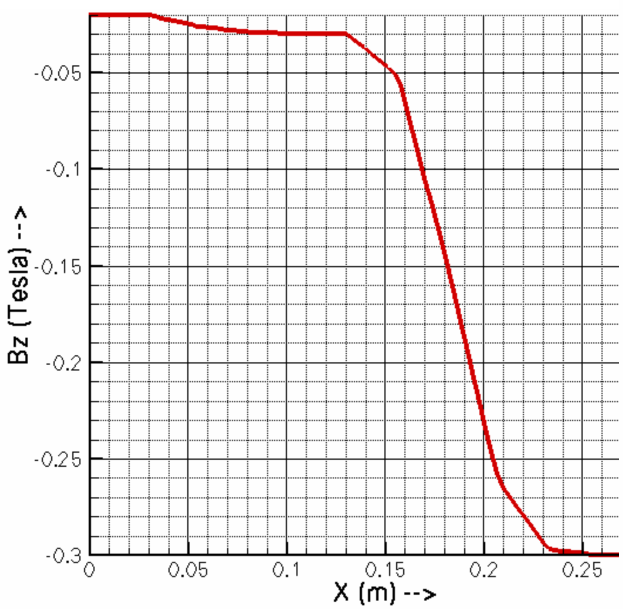




Free surface channel flow

Density ratio 6400, Ga-In-Sn in air
Surface normal magnetic field
Free surface initial thickness 2 mm





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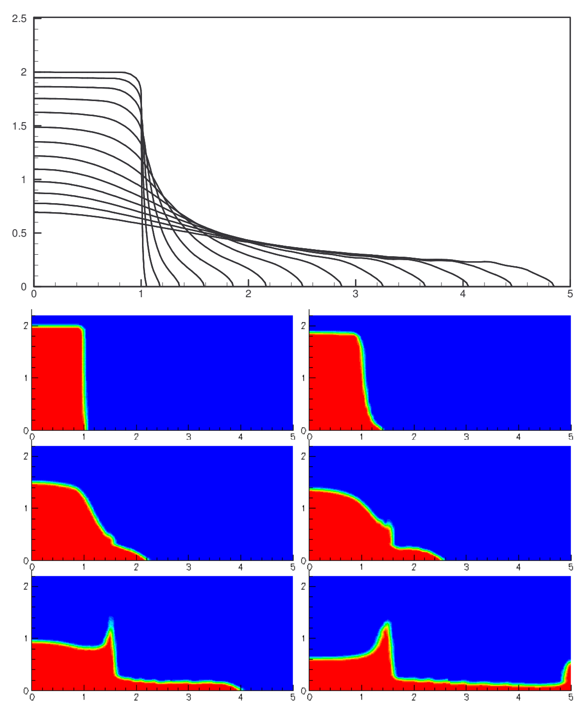
Time-accuracy

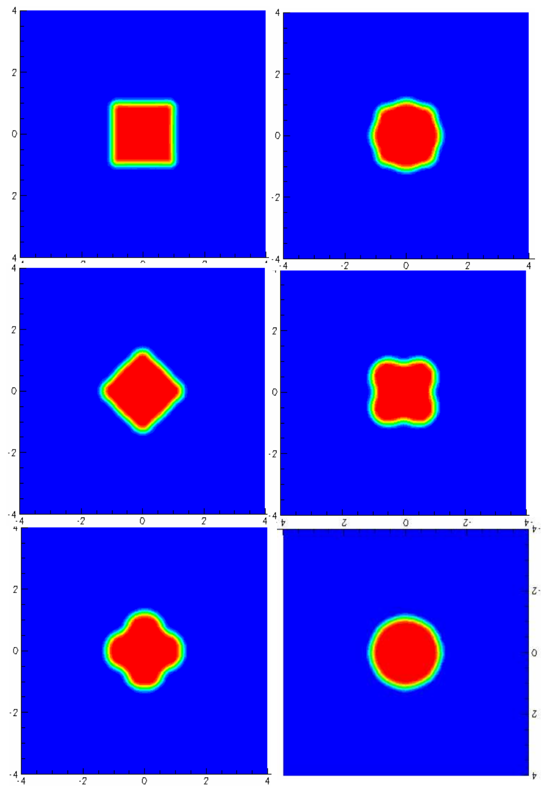
HIMAG uses a second order accurate time-stepping strategy

Sample “canonical” unsteady flow simulations:

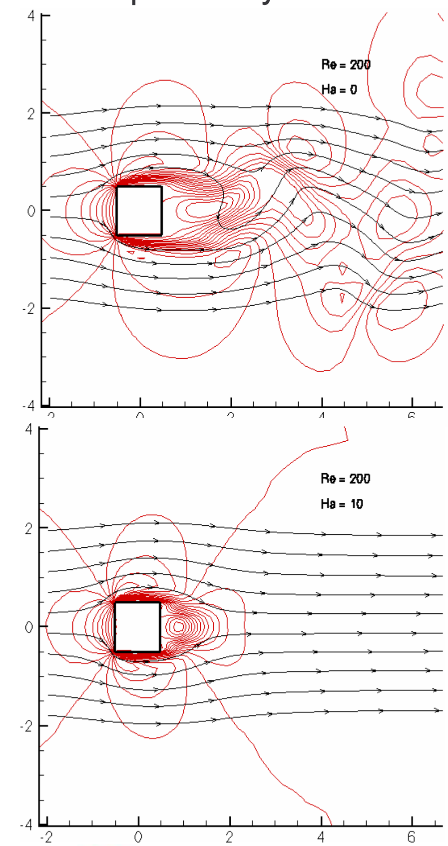
MHD-variant of the “broken-dam” problem



Oscillating droplet

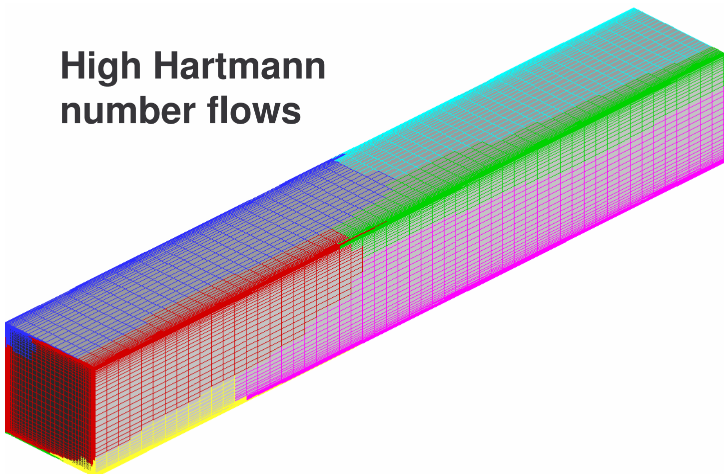


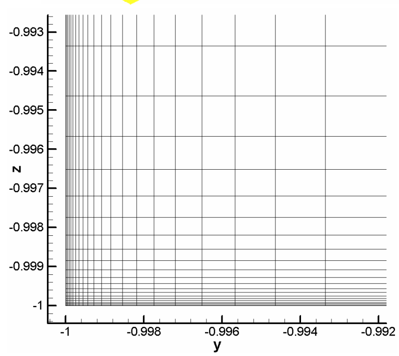
Vortex shedding in MHD flow past a cylinder

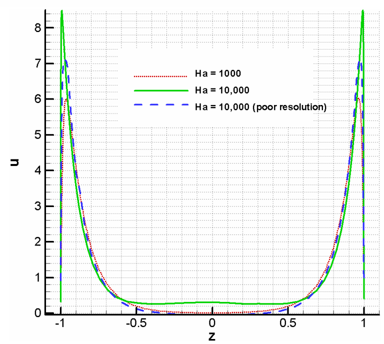


Parallel code execution

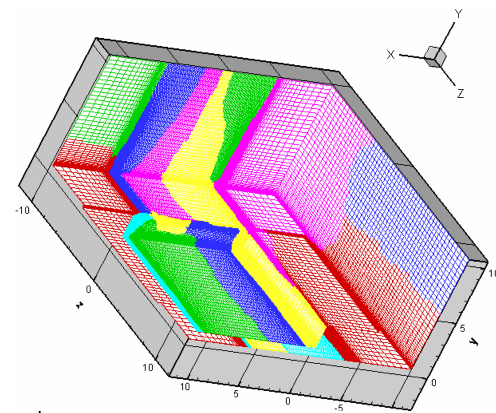
High Hartmann number flows

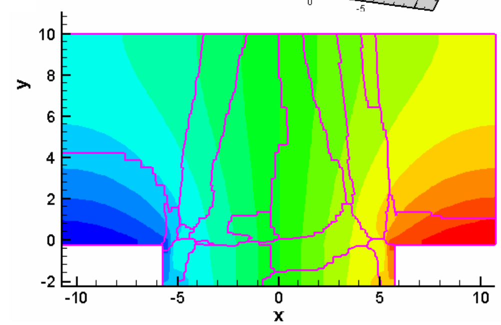


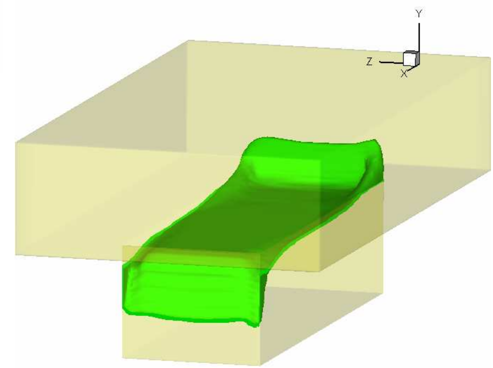




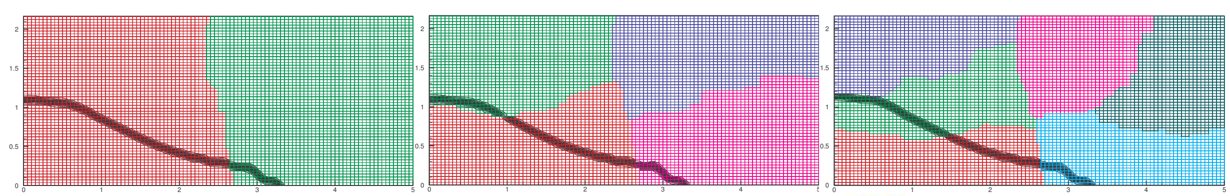
DiMES: MHD sloshing of liquid metal







Validation across multi-processors



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Phase-II: Practical Simulations of two-phase MHD flows with wall effects

The primary goals are:

- To enhance code capabilities in solid wall modeling (including ferromagnetism)
- Development of wall functions, BEM/FVM techniques and investigate other options for high Ha flows
- Development of the B-formulation for HIMAG
- To participate in code applications to engineering design cases in NSTX, DiMES, ITER
- To develop a user group and code dissemination strategy

Phase-I: Development of a canonical approach to liquid metal MHD computation and experiments

- Study flow development criteria numerically – develop numerical metrics for judging code effectiveness, compare with established analytical solutions.
- Develop a database of canonical problems testing each aspect of physical modeling that is sought from HIMAG – perform initial numerical tests.
- Develop simplified interfaces for elementary geometries
- Initiate turbulence and heat transfer modeling efforts
- 3-D code assessment using core-flow analysis codes

Satisfying current conservation

A penalty approach to divergence cleaning (Araseki et al) has been implemented

$$\boldsymbol{J}_t^{k+1} = \sigma\left(-\vec{\nabla}\varphi^{k+1} + \boldsymbol{V}\times\boldsymbol{B}\right)$$

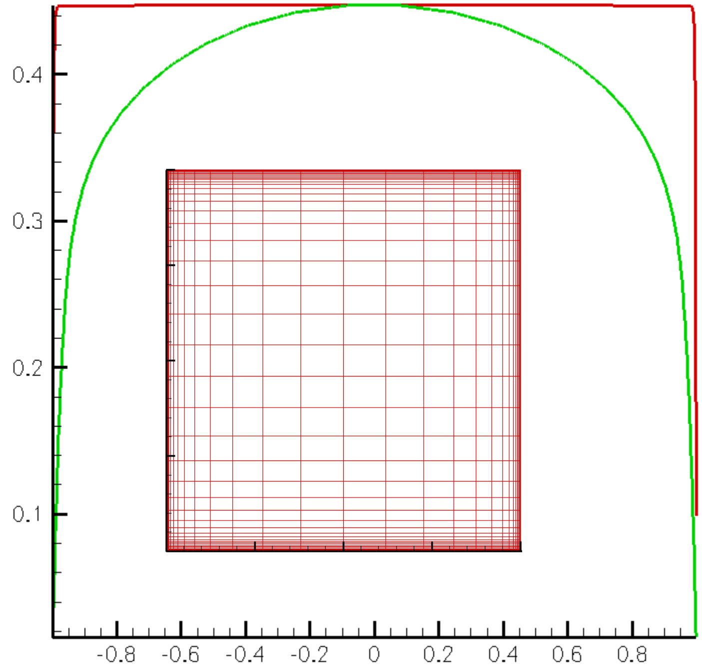
$$\boldsymbol{J}^{k+1} = \boldsymbol{J}^{k} + \omega_J\left(\boldsymbol{J}_t^{k+1} - \boldsymbol{J}^{k}\right), \quad 0 < \omega_J < 1$$

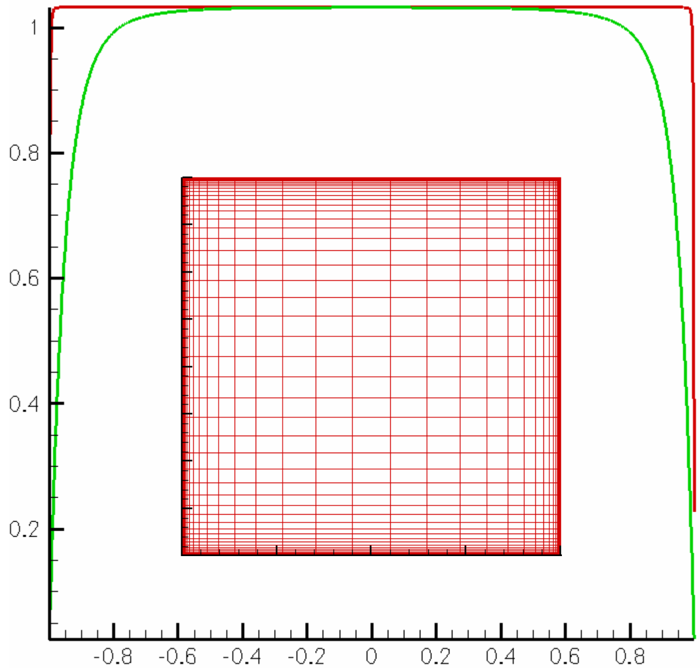
$$\frac{\boldsymbol{J}^{k+1} - \boldsymbol{J}^{k}}{\omega_J} = -\boldsymbol{J}^{k} + \sigma\left(-\vec{\nabla}\varphi^{k+1} + \boldsymbol{V}\times\boldsymbol{B}\right) \qquad \nabla\cdot\boldsymbol{J}^{k+1} = 0$$

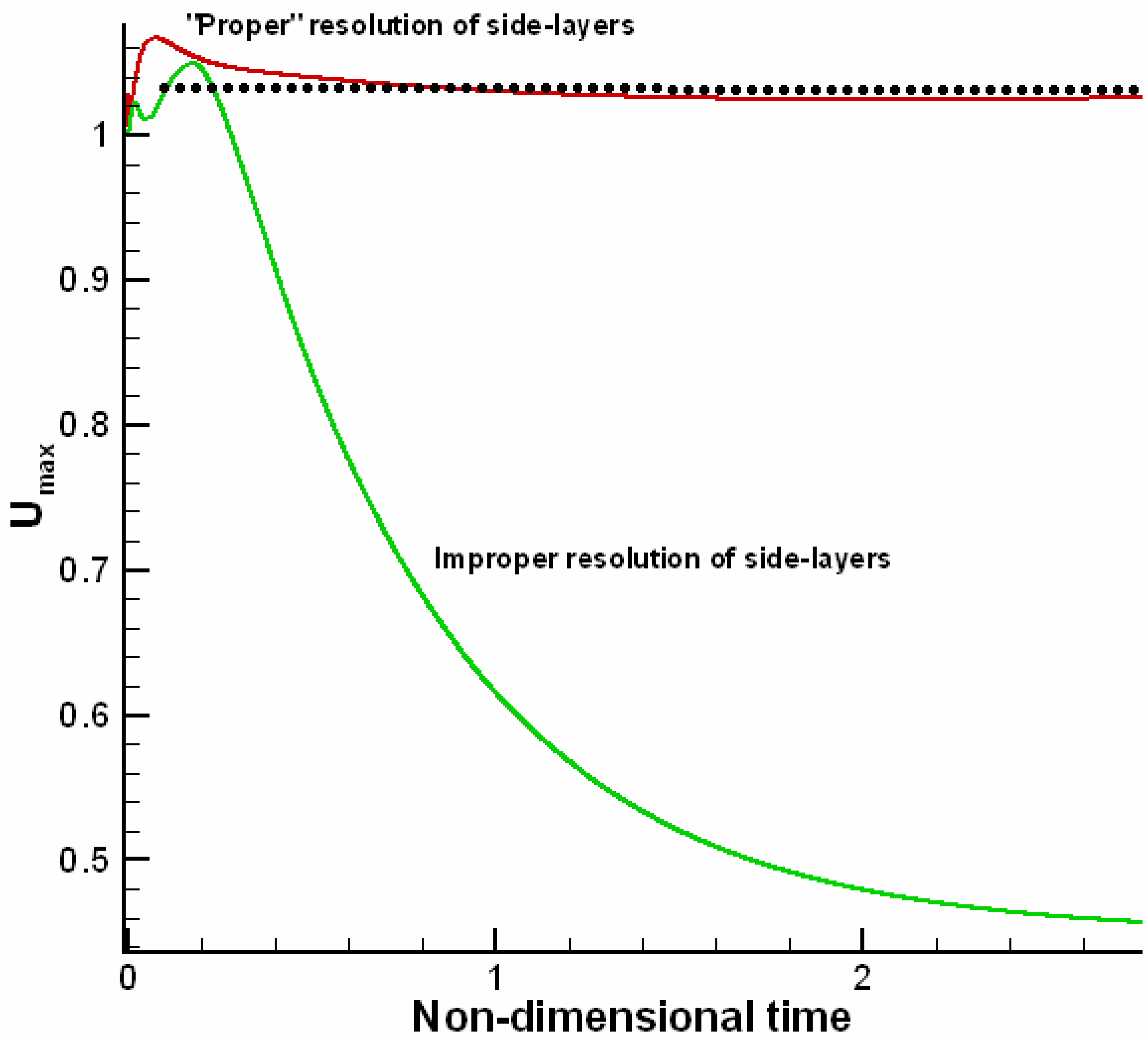
$$\nabla^2\varphi^{k+1} = \vec{\nabla}\cdot\left(\boldsymbol{V}\times\boldsymbol{B}\right) - \frac{1}{\sigma}\left(1 - \frac{1}{\omega_J}\right)\vec{\nabla}\cdot\boldsymbol{J}^{k}$$

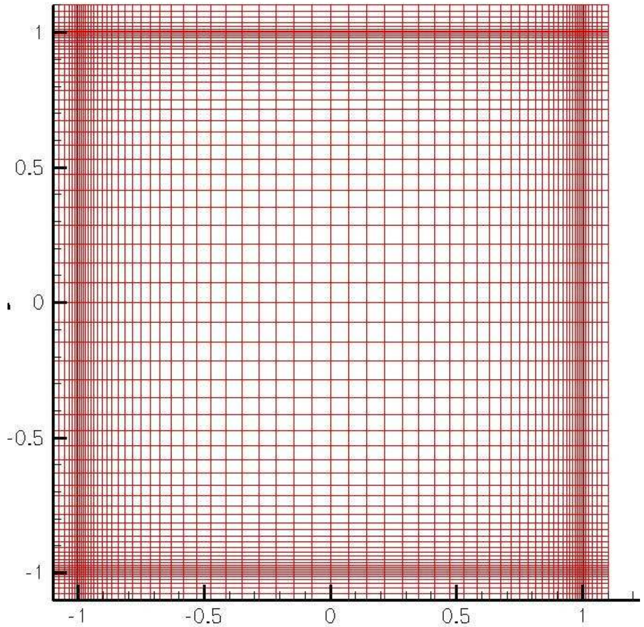
Ha = 500: Effect of mesh and current conservation

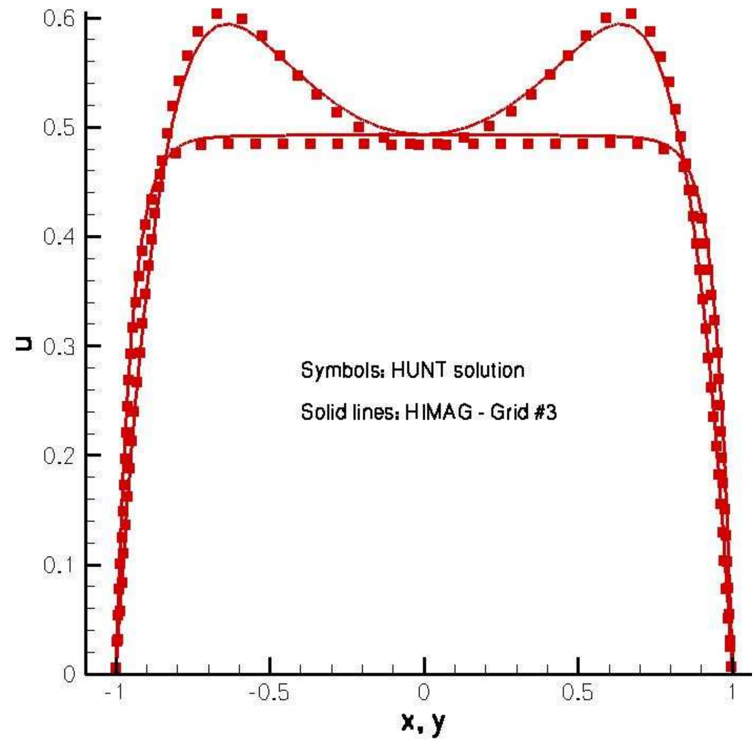
Deliberately under-resolved, 53x46 nodes

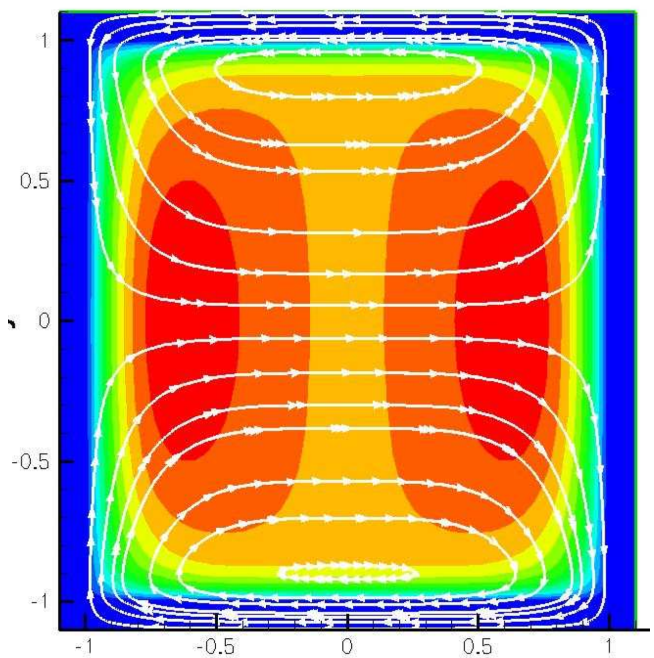












Ha = 20 with conducting walls, fully developed flow.

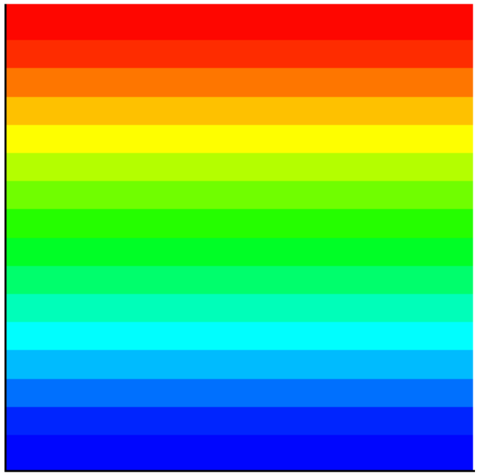
Comparison with Hunt's solution.

$c_w = 0.1$

Two formulations for MHD are now available in HIMAG:

Inductionless “φ” formulation

$$\nabla \cdot (\sigma \nabla \varphi) = \nabla \cdot (\sigma V \times B)$$



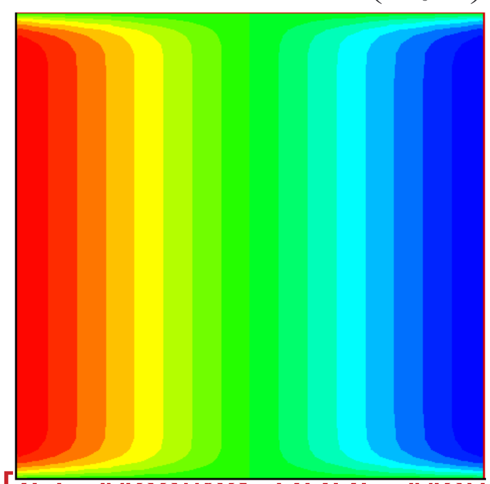
An inductionless and an induced B-field formulation

These are coupled with a pre-processor which uses the Biot-Savart law to reconstruct a magnetic field from a system of sources.

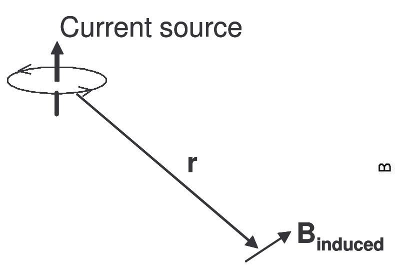
An arbitrary set of conducting walls can be used in the computation.

Induced field “B” formulation

$$\frac{\partial \vec{B}}{\partial t} - \vec{\nabla} \times (\vec{V} \times \vec{B}) = -\vec{\nabla} \times \left(\frac{\vec{\nabla} \times \vec{B}}{\mu_0 \sigma} \right)$$

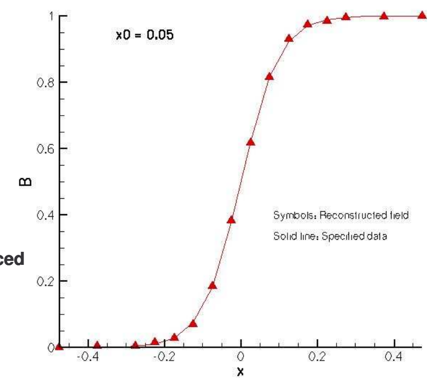


Reconstruction of div and curl free B-fields from measurements

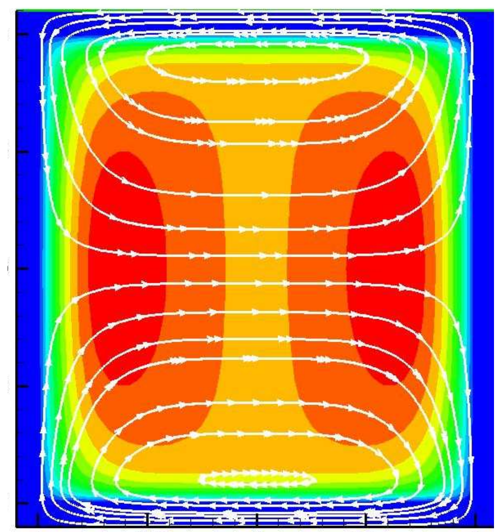


$$\delta \vec{B} = \alpha \frac{\vec{r} \times \delta \vec{I}}{r^3}$$

Biot-Savart law



Current computation with conducting walls



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The **B**-formulation

Integrating over a control volume, and using the Euler implicit scheme, we get:

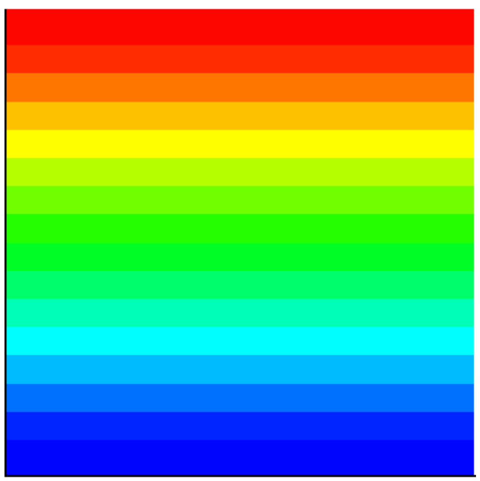
$$\frac{\boldsymbol{B}_i^{n+1}-\boldsymbol{B}_i^{n}}{\Delta t}=\frac{1}{\Omega}\oint_{\partial\Omega}\left\{\hat{\boldsymbol{n}}\times\left(\vec{\boldsymbol{V}}\times\vec{\boldsymbol{B}}\right)\right\}_i^{n+1}\boldsymbol{ds}-\frac{1}{\Omega}\oint_{\partial\Omega}\frac{1}{\mu_0\sigma}\left\{\hat{\boldsymbol{n}}\times\left(\vec{\nabla}\times\vec{\boldsymbol{B}}\right)\right\}_i^{n+1}\boldsymbol{ds}$$

The finite volume expression is then written as:

$$\boldsymbol{B}_i^{n+1}=\boldsymbol{B}_i^{n}+\frac{\Delta t}{\Omega}\sum_{faces}\left\{\hat{\boldsymbol{n}}\times\left(\vec{\boldsymbol{V}}\times\vec{\boldsymbol{B}}\right)\right\}_i^{n+1}\Delta s-\frac{\Delta t}{\Omega}\sum_{faces}\frac{1}{\mu_0\sigma}\left\{\hat{\boldsymbol{n}}\times\left(\vec{\nabla}\times\vec{\boldsymbol{B}}\right)\right\}_i^{n+1}\Delta s$$

Inductionless “φ” formulation

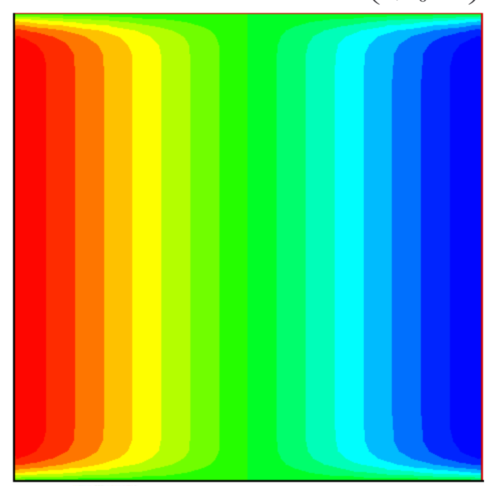
$$\nabla\cdot(\sigma\nabla\varphi)=\nabla\cdot(\sigma V\times B)$$



Ha = 500 solution
Using phi and B formulations

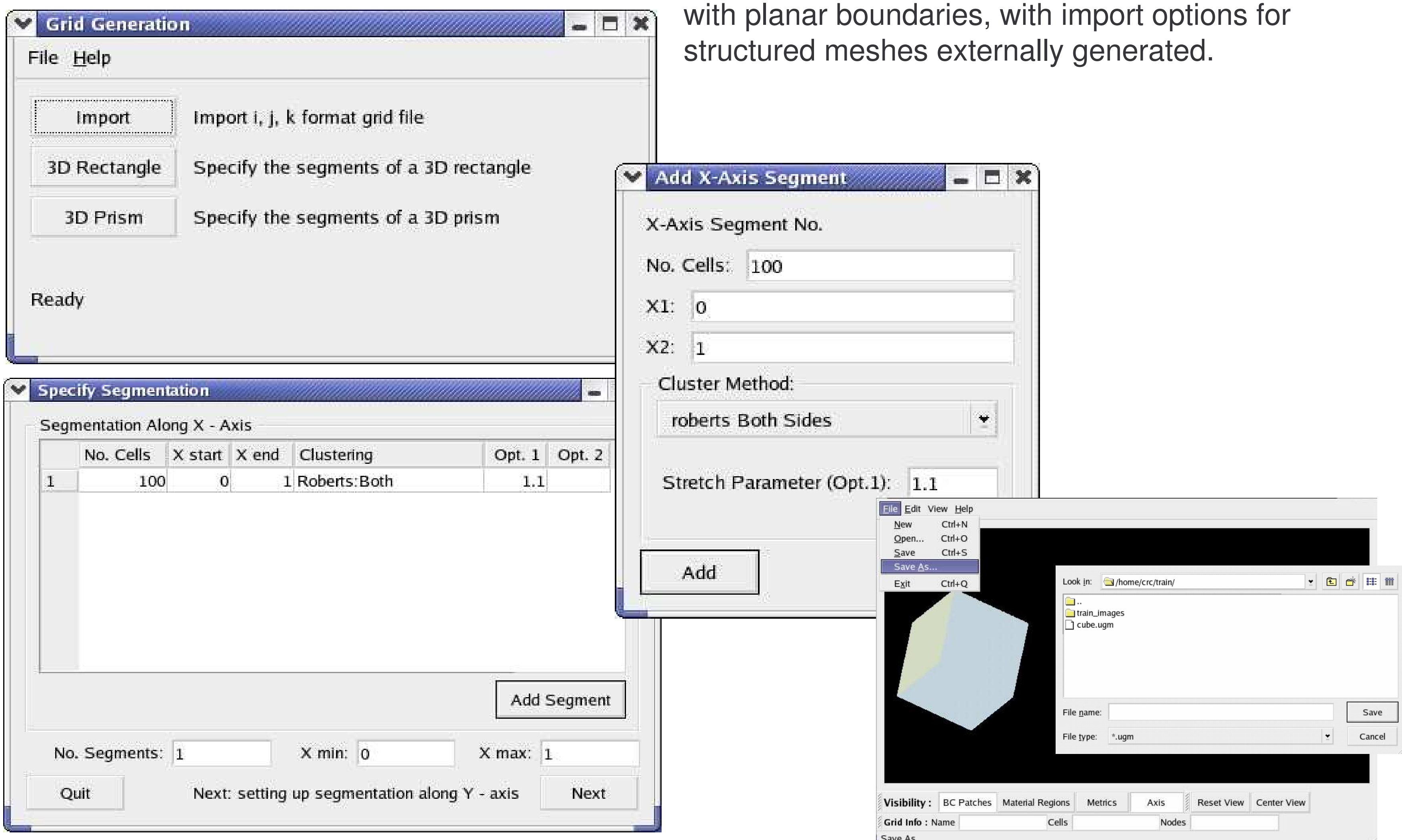
Induced field “B” formulation

$$\frac{\partial\vec{\boldsymbol{B}}}{\partial t}-\vec{\nabla}\times\left(\vec{\boldsymbol{V}}\times\vec{\boldsymbol{B}}\right)=-\vec{\nabla}\times\left(\frac{\vec{\nabla}\times\vec{\boldsymbol{B}}}{\mu_0\sigma}\right)$$

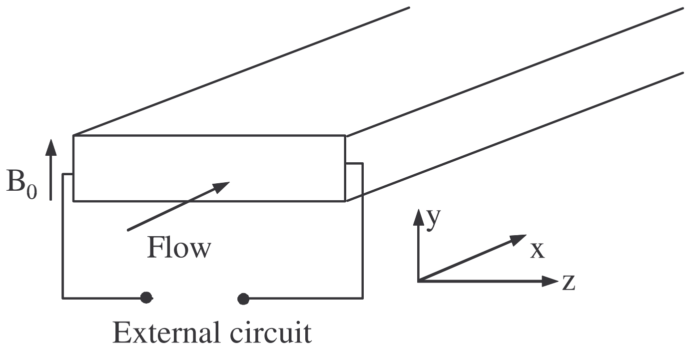


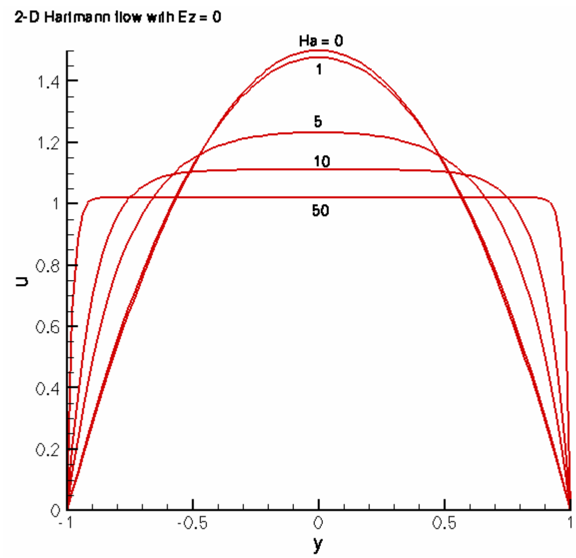
Easy problem setup for simple geometries:

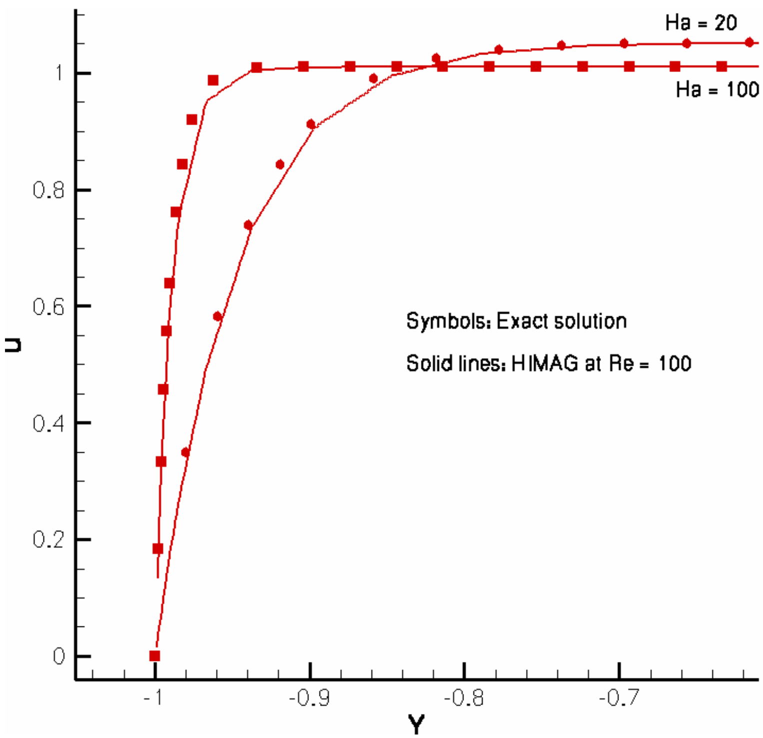
The illustration shows a GUI which assists the user to setup simple meshes for MHD calculations with planar boundaries, with import options for structured meshes externally generated.

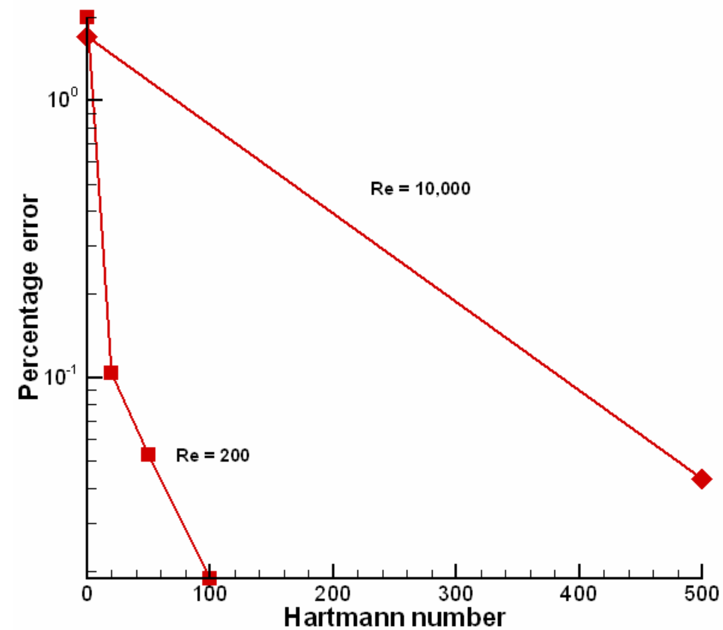


2-D Hartmann channel flow – a preliminary study of numerical errors









Time-step limitation: Theoretical and observed

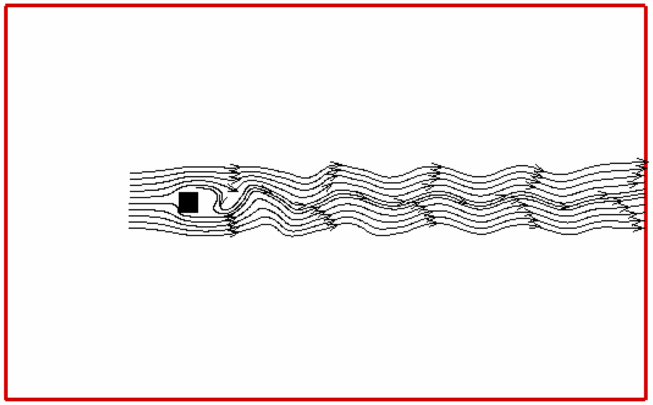
$$\Delta t_c = \min\left(\frac{h}{|u|}\right)$$

$$\Delta t_{viscous} = \min\left(\frac{3}{14}\frac{\rho \operatorname{Re} h^2}{\mu}\right)$$

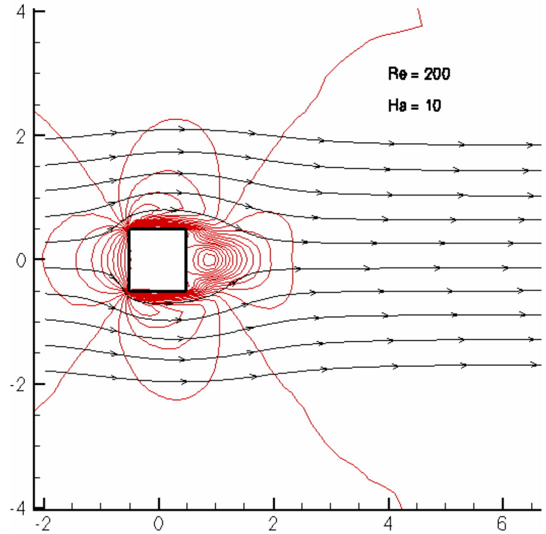
Re	h	Δt_c	Δt_{visc}	Δt_{HIMAG}
100	0.05	0.05	5.36	0.25
100	9.7e-5	9.7e-5	2.0e-5	1.0e-4
10	0.05	0.05	0.0536	0.05
10	3.0e-3	3.0e-3	1.9e-4	8.0e-4
10	9.7e-5	9.7e-5	2.0e-7	3.0e-7

Benchmark cases:

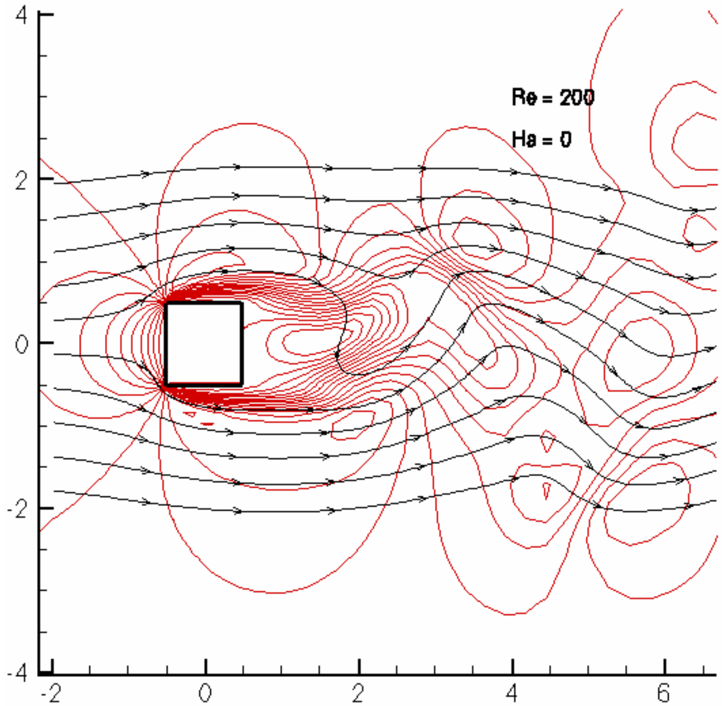
Vortex shedding in MHD flows

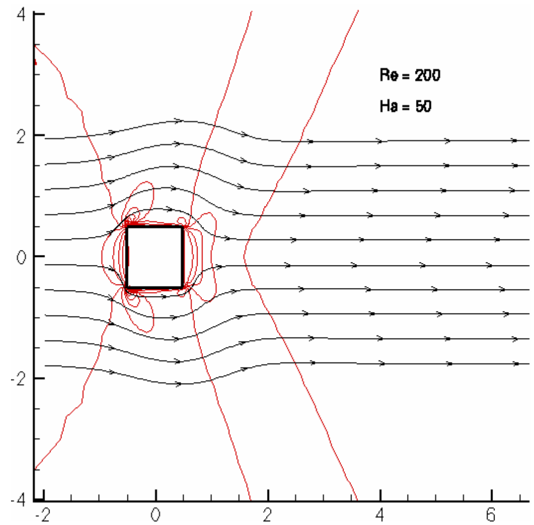


2-D flow past a square cylinder with flow-aligned B



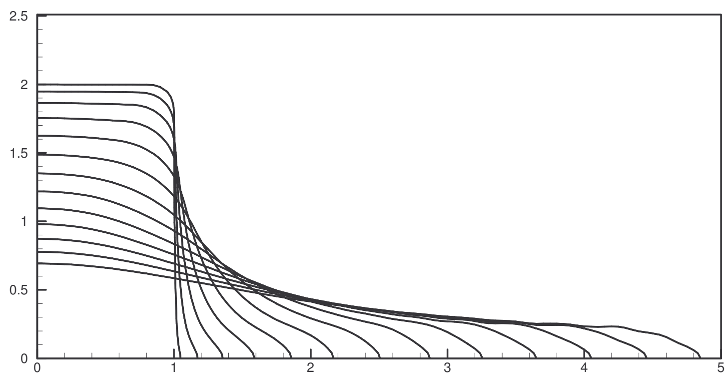
N = 0 (left), 0.5 (above), 12.5 (below) with u-contours

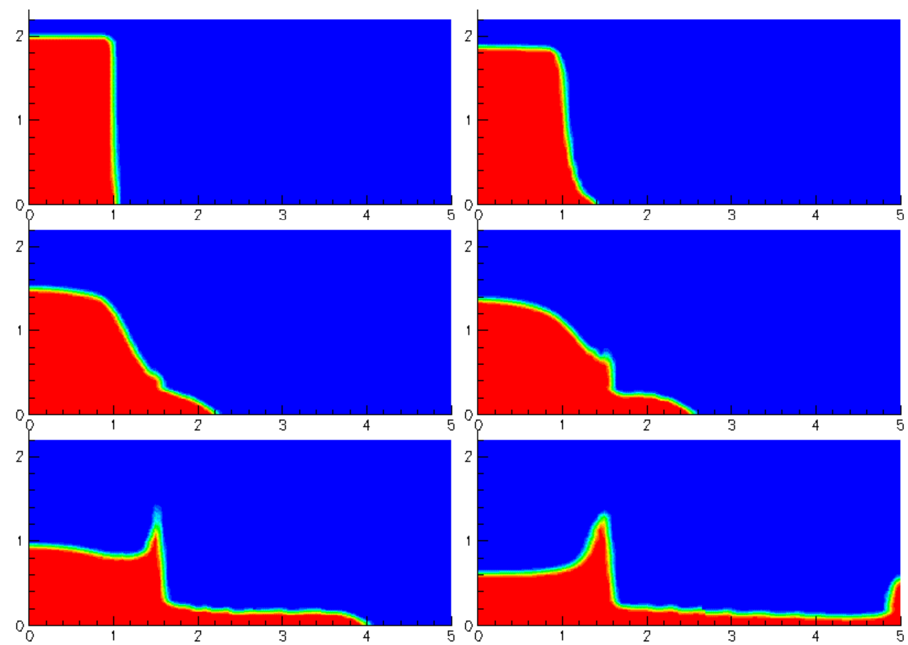




Benchmark cases:

An MHD equivalent to the "broken dam" problem





B_y held constant in the region x > 1.5 and y > 0.3 and linearly tapered away from this region. The Hartmann number is 316, based on a characteristic length of 1.

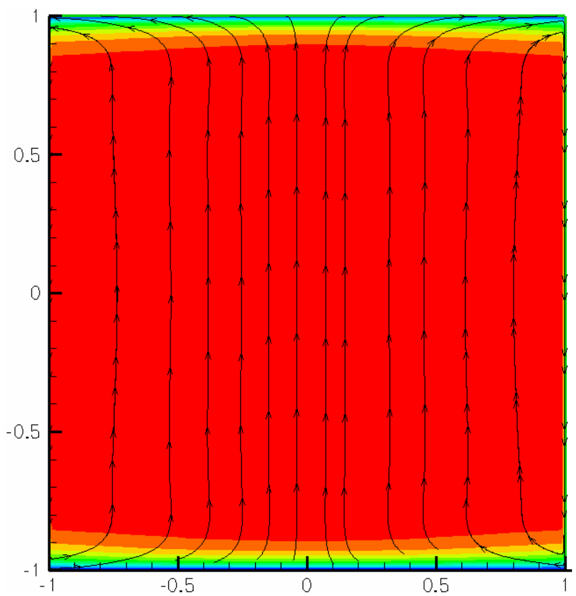
$$\mathrm{Re} = \frac{\rho L \sqrt{2gL}}{\mu}$$

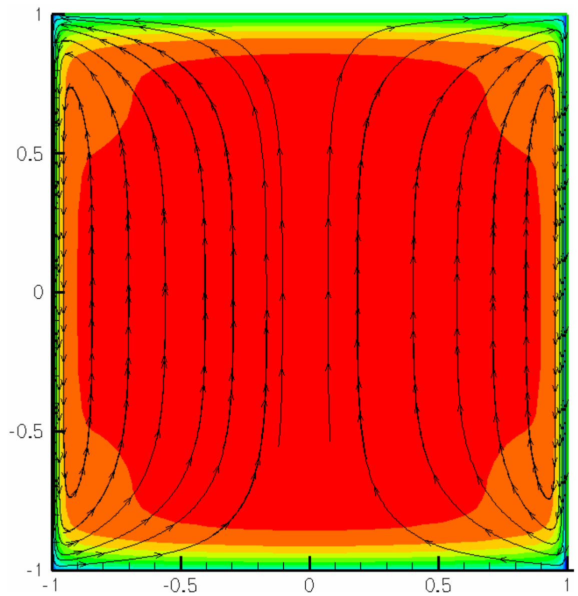
L = 1, and g is the acceleration due to gravity (also 1). The coefficient of viscosity was selected as 0.001. With this combination, we get a Reynolds number of about 1414. The interaction parameter can be estimated to be about 71.

Turbulence Modeling

Ha = 500, Re = 10,000

Laminar 2-D turbulent profile





While the k-ε model code is being readied, We are experimenting with simpler models for high Ha: 2-D Turbulence

$$\frac{\nu_{ty}}{\nu} = \frac{a}{b} Re_* [1 - Exp\{-\sqrt{Ha}(a/b - |\, y/b \,|)\}]$$ (Messadek and Moreau)

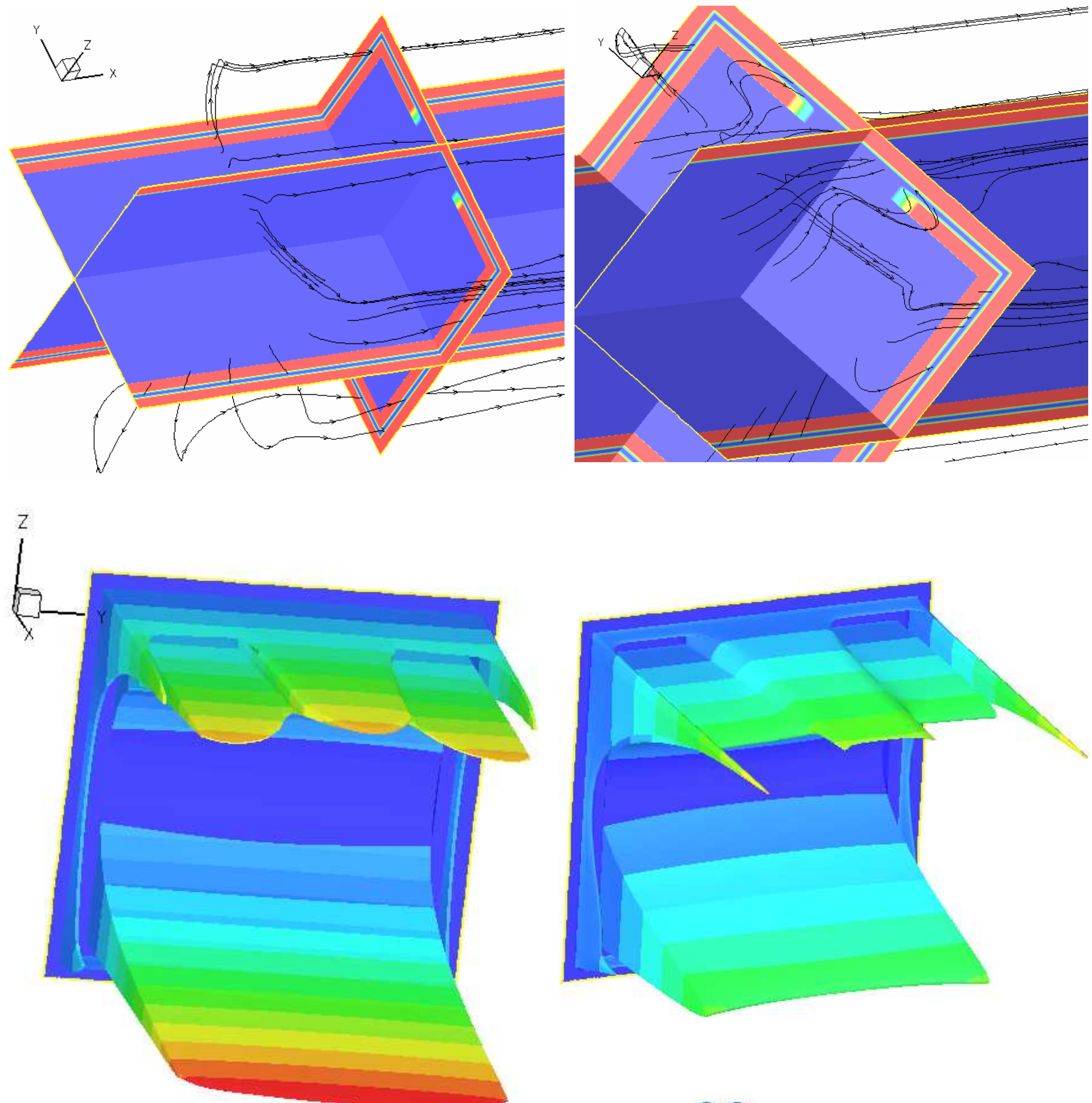
FUSION-RELATED APPLICATIONS,

ISSUES AND ENHANCEMENTS

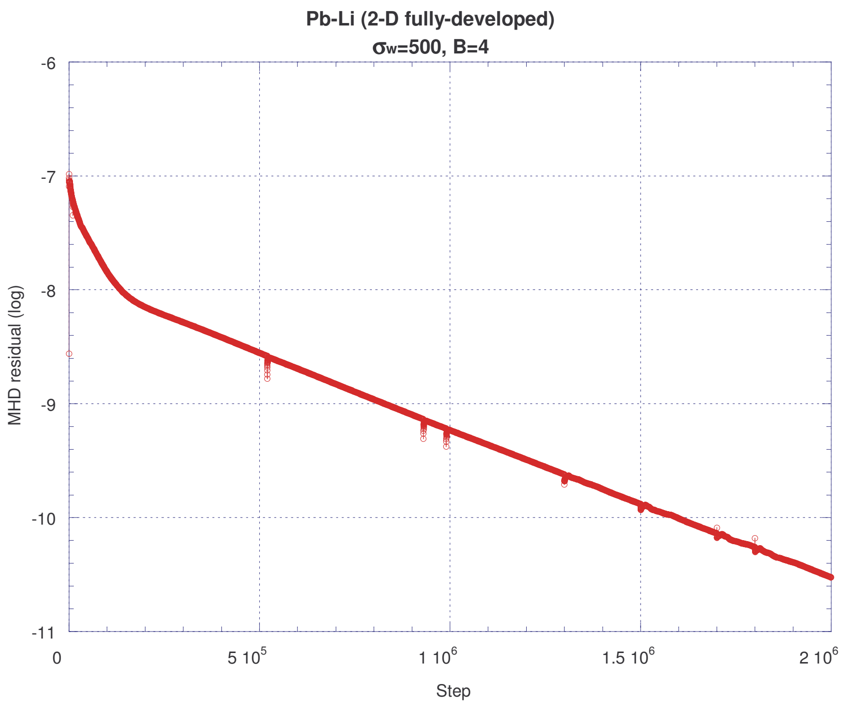
Dual Coolant FCI simulations

Flow uniqueness:

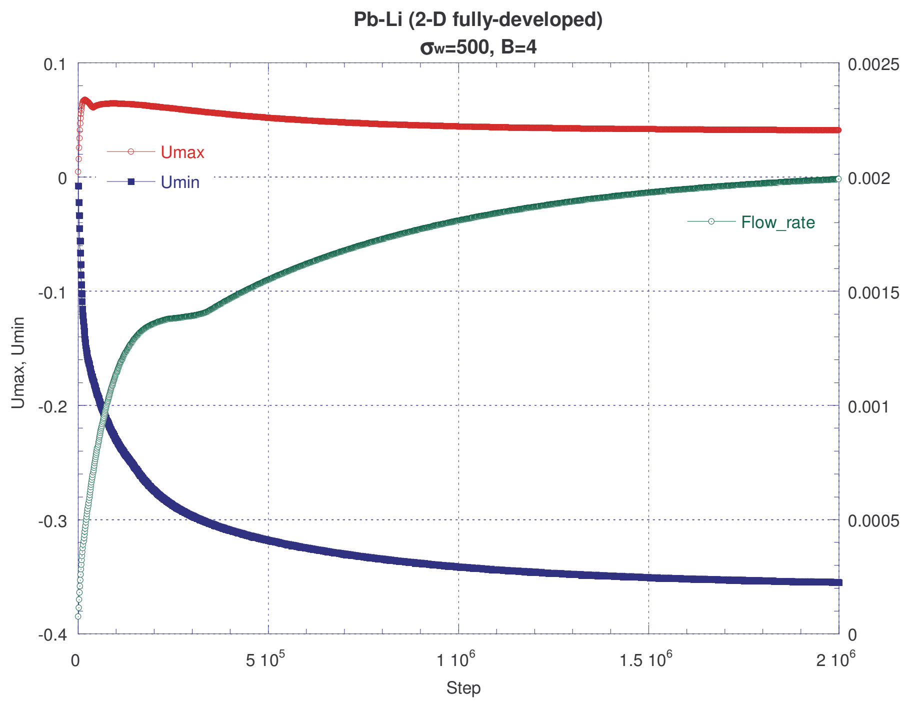
Are there closed channel flows of ITER interest which do not satisfy the assumptions of Hunt's (1969) uniqueness theorem?



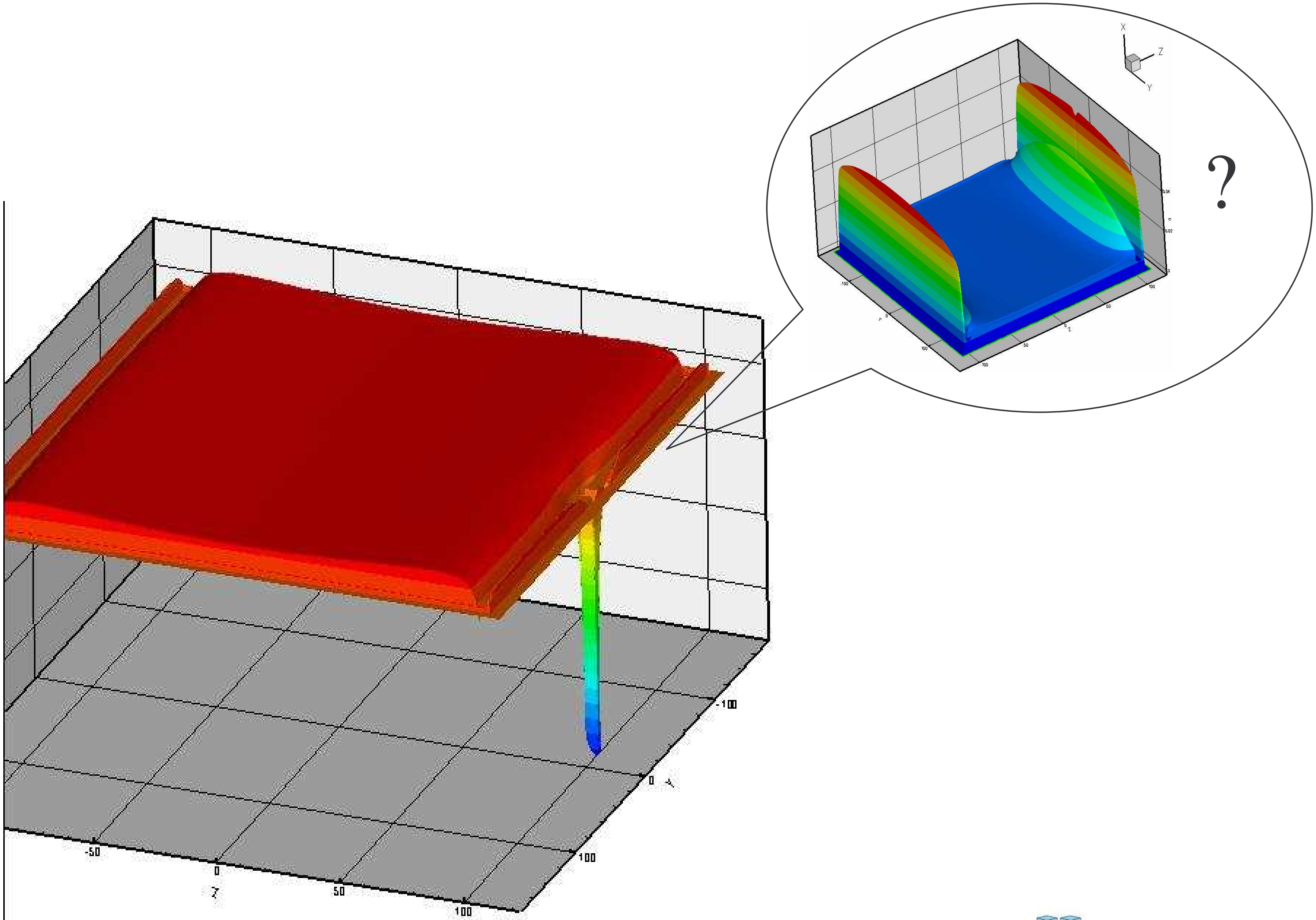
σ_w=500

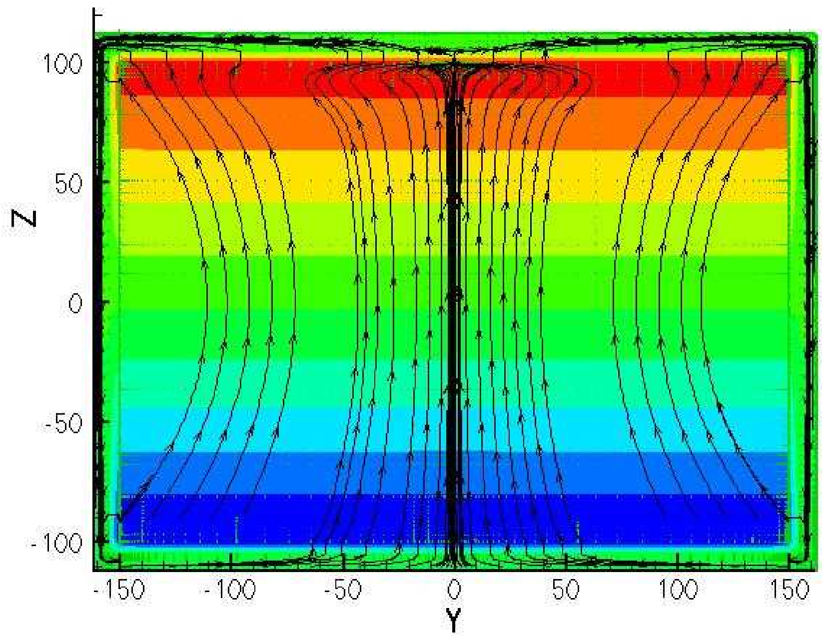


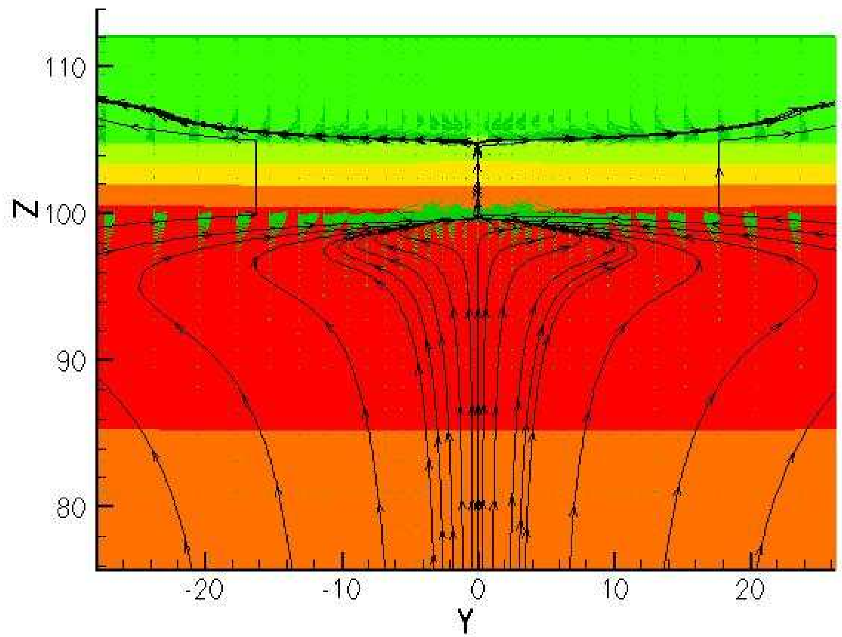
Pb-Li (2-D fully-developed)
σw=500, B=4
MHD residual (log)
-6
-7
-8
-9
-10
-11
0
5 10^5
1 10^6
1.5 10^6
2 10^6
Step



Pb-Li (2-D fully-developed)
σw=500, B=4
Umax
Umin
Flow_rate
Umax, Umin
0.1
0
-0.1
-0.2
-0.3
-0.4
0.0025
0.002
0.0015
0.001
0.0005
0
0
5 10^5
1 10^6
1.5 10^6
2 10^6
Step


?
-50
0
Z
50
100
-100
0
Y
100





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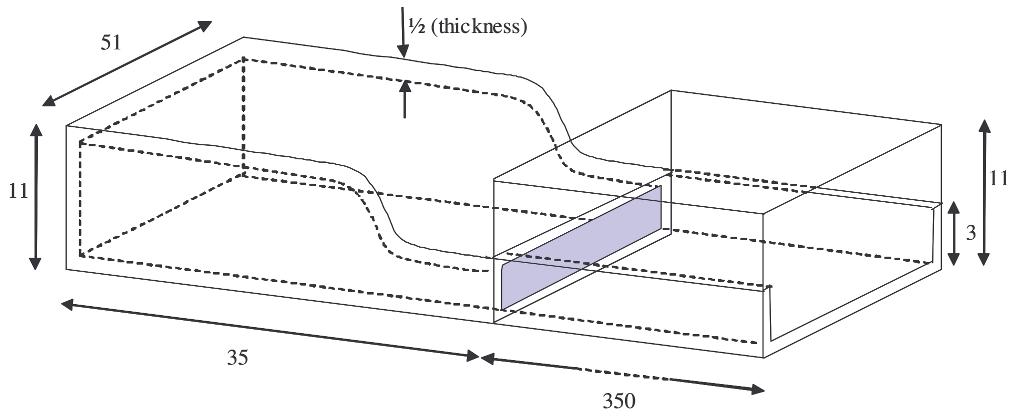
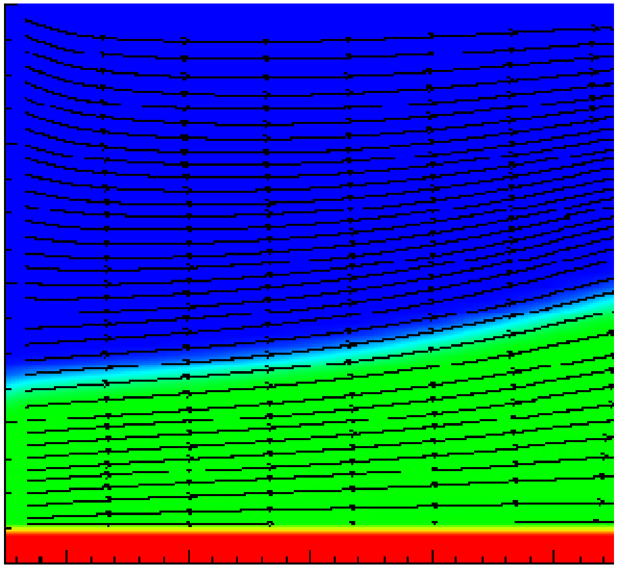
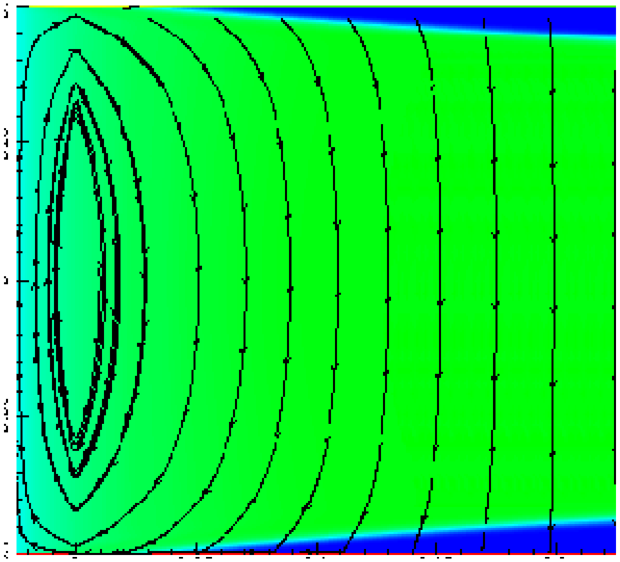
MTOR simulation using HIMAG:

Multi-material, free surface flow simulation in non uniform magnetic field (NSTX divertor region surface normal field component)





The free surface of the liquid metal flow. Inlet velocity is 3.0*m/s*, Inlet film thickness is 2.0*mm*. The fluid under study is Ga alloy, flowing under argon atmosphere (density ration 4000) on a stainless steel substrate (electrical conductivity 1/3 that of liquid metal). The conducting wall thickness is 0.5*mm*.

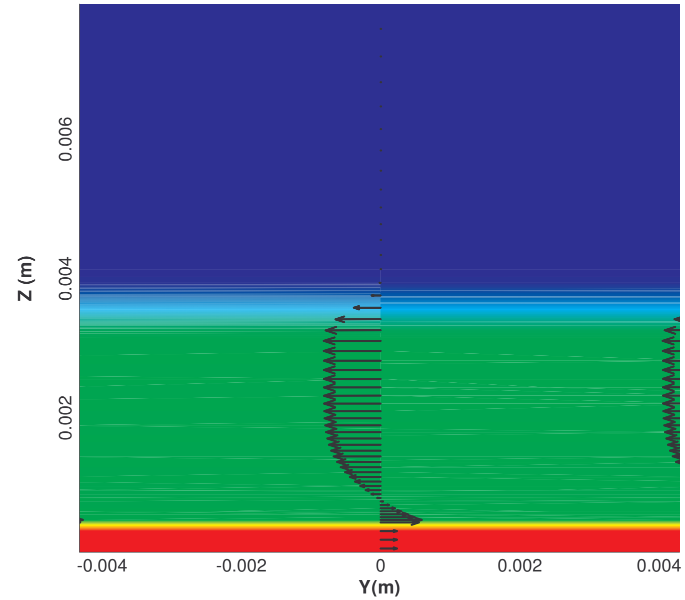


Stream-wise flow (above)
and
Span-wise current distribution
(below)

Issues in flow detachment:

➢ Large spread of cell size required by high Ha considerations introduces errors in interface capture.

Fix: A “dynamic” model of free surface resolution wherein the level set function is smoothed locally instead of globally.

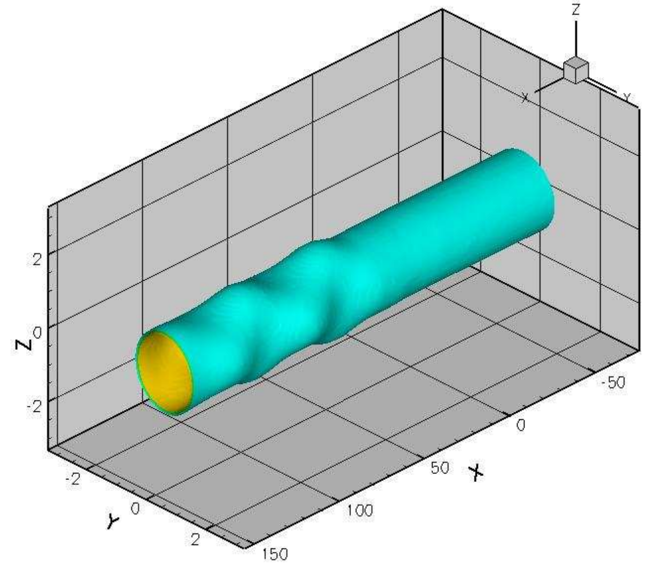


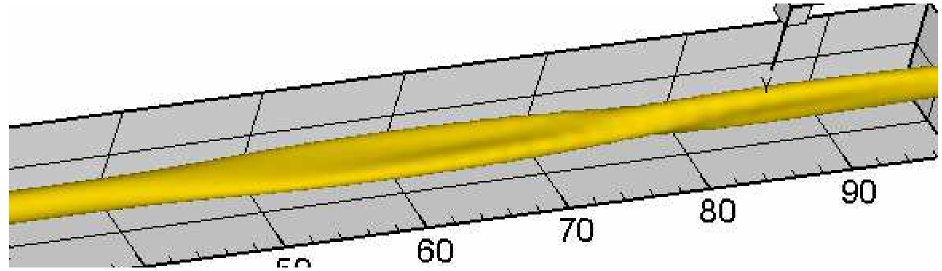
➢ Contact angle / wetting can influence the formation and advancement of detached flow.

Fix: Contact angle BCs are being investigated

Jet flow under a 3-component applied B-field:

The warping of a liquid metal jet under a B-field with sharp spatial gradients is studied. The free surface can deform sizably.

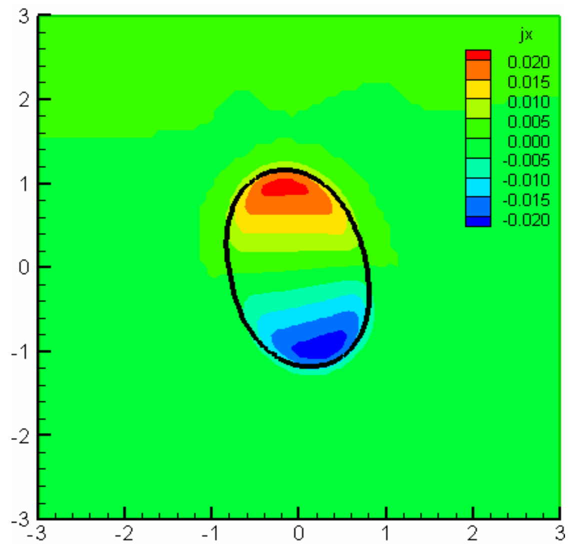


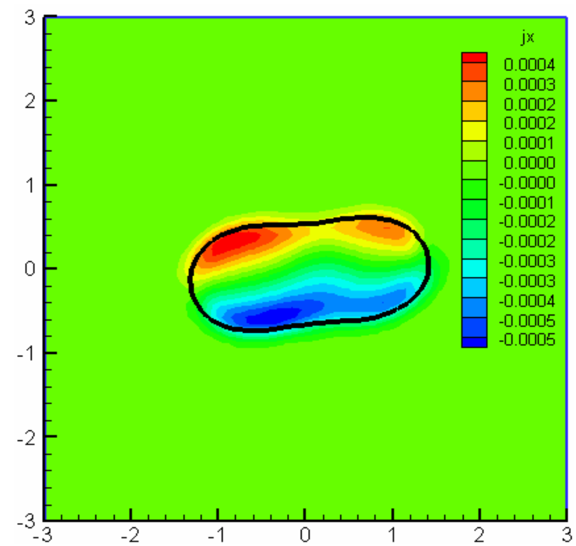


Jet at the real aspect ratio, and exaggerated aspect ratio on the left.

Needs: high spatial resolution to model jet pinching / thinning

Below: Axial current density at two cross sections





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HIMAG – future work

- Incorporation of arbitrary magnetic properties in walls – in particular, ferromagnetic effects.
- Wall functions in accelerating convergence of high Hartmann number flows
- A hybrid Finite Volume – Boundary Element method
- Systematic benchmarking and validation.
- Assist simulation efforts and build a user community, develop user friendly interfaces for data import/export from other solvers
- *Natural convection, Tritium and Heat transfer modeling*
- *Turbulence model development: Two equation model for low Hartmann numbers and a zero-equation model for high Hartmann numbers*